The Engineer and the Mathematician

By Liza Faust

I was ten or eleven I think. During a summer break visiting my father we all trooped out to visit Aunt Helen and Uncle Mike. I walked past a table where Uncle Mike and Dad were arguing something arcane, stopped, looked right at them and asked, “So what’s the difference between an engineer and a mathematician?”

They both stared. Dad took the first turn. “Well Liza,” he said flipping his bangs and raising one eyebrow in his ‘I love to explain things’ style, “an engineer is someone who takes esoteric knowledge and puts it to Good and Practical Use making things that Serve Humanity.”

I turned to Uncle Mike and suddenly he became backlit, there was a shiny glow around his head, he stretched out his arms waist height, palms up and I could hear the choir of angels in the background singing “AaaaahhhhHHHH!” as he said, “A mathematician deals only in the purest truths of the universe, sometimes without involving numbers at all, and often proving ideas fundamental to the way our world works...”

… honestly I don’t remember all he said about it, but it was a deeply religious experience.

Go on, ask me what my degree is in.

From Uncle Mike: Excellent, excellent! Dang, I was beautifully backlit in those days! Gives me a warm feeling all over!

OK, now eventually you are probably going to have to cogitate and segue into the ascension of Vector Angle ... but don't step on those purest truths ...

… Like the last digit of pi as discussed by Marc Clamage on Quora.com:

> My son once bragged to his friend (facetiously) that he knew all the digits of pi. His friend’s dad said, “Oh yeah, what’s the last digit?” My son responded, “3. It’s a palindrome.”
>
> Apparently he got that line from some web comic[1] but props for the zingy response nonetheless.
>
> I like Buddha Buck’s response for making the points that (a) mathematical statements of the form “if <False> then X” are true for any proposition X, and (b) there is an important distinction between a real number and a system for representing it.

[1] http://abstrusegoose.com/440

While I agree with point (a), it's only fair to acknowledge that we often engage in speculative counterfactuals in daily life. "If I hadn't misread the map, I would have been on time" makes a perfectly sensible statement about likely events in a fictional world where I read a map correctly. To say instead "I would have been on thyme" is mathematically equivalent, but wouldn't make a lot of sense unless the map is of an herb garden.

In mathematics, speculative counterfactuals tend to produce logical contradictions. This is useful for proving that a counterfactual is, in fact, false. If we are lucky we may establish that our "counterfactual" is actually independent of our axiomatic system: this is how non-Euclidean geometry was discovered. But if we do prove that our counterfactual is false, then there is little point in engaging in further ruminations about the nature of that fictional world we have conjured up. In such a world, all statements are true. And false. So in that sense, if a decimal representation of pi terminated, it is equally true that it would terminate in 0, 1, 2, 8, q, %, ?, or the Pope.

As for point (b), on the one hand you could say, "of course he means the decimal representation of pi. Why quibble?" But on the other hand, conflating a real number with its decimal representation is what leads to confusion about the notorious 1 = 0.999… issue. So it is worth making the distinction.

When we write "3.14159…" we are indicating a certain real number, implicitly invoking a fair amount of machinery to do so. First, the "…" implies that that there is an infinite sequence of digits that follow, and that the writer and reader of the "…" understand which sequence of digits is meant. We convert this sequence of digits into a (non-decreasing) sequence of rational numbers: (3, 3.1, 3.14, 3.141, 3.1415, 3.14159, …). Such a sequence may then be used to indicate a real number. Under the Dedekind model of real numbers, it indicates the union of all rational numbers less than 3, all rational numbers less than 3.1, and so on. Under the Cauchy model, it indicates the equivalence class of the sequence.

Because people readily accept that XLII, forty-two, and 42 are representations of an integer (rather than the integer itself), they also seem willing to accept that 3.14159… and pi are both just representations of a certain real number. But the problem is that if you call something a "representation of X" it raises the question of what X itself is. An integer is an intuitive notion, but a real number is not. The Dedekind and Cauchy models both capture the essence of real numbers in different ways (i.e., via an order vs. a metric). They show how real numbers "fill in all the holes between rational numbers" but why there is no real number "in the hole between 0 and the positive rationals."

Without an easily grasped notion of real numbers — the somethings that the representations 3.14159… and 0.999… point to — it is all too easy to

> backslide into treating the representations as the numbers themselves. Through the hazy reasoning this engenders, we might argue that 1 - 0.999… is some tiny positive number, a one infinitieth, if you will, which is only slightly larger than zero. Its decimal representation has an infinite number of zeros followed by a 1 (in the infinitieth place). What else can we conjure up in this hazy dreamworld? Oh right, pi: it starts 3.14159…, continues for an infinite number of digits, and then ends in …951413. How do I know? I declare myself the Pope and invoke papal infallibility.

And I love this little story, offered as a follow-up to Clamage's essay, which shows how easy it is to underestimate the power of infinity:

> I once answered a question about proving the existence of God with the comment that, if someone were to find a section of pi that, when converted to hexadecimal proved to be text that stated "Hi! I'm God. I do exist so that settles that," I would consider that sufficient proof of the existence of God. Someone pointed out in a comment that, by definition, a section of numbers convertible to that ***can and must exist*** in pi, along with the complete works of Shakespeare and the lost works of Plato.
>
> Oops.